

1. Solve the equation

$$3^x - 5^y = z^2$$

in positive integers.

2. Let MN be a line parallel to the side BC of triangle ABC , with M on the side AB and N on the side AC . The lines BN and CM meet at point P . The circumcircles of triangles BMP and CNP meet at two distinct points P and Q . Prove that $\angle BAQ = \angle CAP$.

3. A 9×12 rectangle is partitioned into unit squares. The centres of all the unit squares, except for the four corner squares and the eight squares sharing a common side with one of them, are coloured red. Is it possible to label these red centres $C_1, C_2, \dots, C_{96}$ in such a way that the following two conditions are both fulfilled

- (i) the distances $C_1C_2, C_2C_3, \dots, C_{95}C_{96}, C_{96}C_1$ are all equal to $\sqrt{13}$,
- (ii) the closed broken line $C_1C_2 \dots C_{96}C_1$ has a centre of symmetry?

4. Denote by S the set of all positive integers. Find all functions $f : S \rightarrow S$ such that

$$f(f(m)^2 + 2f(n)^2) = m^2 + 2n^2, \quad \text{for all } m, n \in S.$$

Time allowed: 4 hours and 30 minutes.
Each problem is worth 10 points.